Usual probability distributions

Discrete distributions

Name, parameters	$X(\Omega)$	$\mathbb{P}(X=k)$	Expected value	Variance
Bernoulli $X \sim \mathscr{B}(1,p)$	$\{0\,,\,1\}$	p if $k=1$ $1-p$ if $k=0$	p	$p(1-p)$
binomial $X \sim \mathscr{B}(n,p)$	$[\![0,n]\!]$	$\binom{n}{k} p^k (1-p)^{n-k}$	np	$np(1-p)$
hypergeometric $X \sim \mathcal{H}(N,n,m)$	$\subset [\![0,n]\!]$	$\frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}}$	$\frac{nm}{N}$	$\frac{n\,\frac{m}{N}\left(1-\frac{n}{N}\right)(N-n)}{N-1}$
geometric $X \sim \mathscr{G}(p)$	$\mathbb{N}^*$	$p(1-p)^{k-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson $X \sim \mathscr{P}(\lambda)$	$\mathbb{N}$	$\mathrm{e}^{-\lambda}\,\frac{\lambda^k}{k!}$	λ	λ

Continuous distributions

Name, parameteres	$X(\Omega)$	Definition	Density $f(t)$	Expected value	Variance
uniform $X \sim \mathscr{U}\big([a,b]\big)$	$[a,b]$		$\frac{1}{b-a}$ if $t \in [a,b]$ 0 otherwise	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
exponential $X \sim \mathscr{E}(\lambda)$	$[0,+\infty[$		$\lambda\,\mathrm{e}^{-\lambda t}$ if $t \geq 0$ 0 if $t<0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
normal $X \sim \mathcal{N}(\mu,\sigma^2)$	$\mathbb{R}$		$\frac{1}{\sqrt{2\pi\sigma^2}}\,\mathrm{e}^{-\frac{(t-\mu)^2}{2\sigma^2}}$	μ	σ^2
standard normal $X \sim \mathcal{N}(0,1)$	$\mathbb{R}$		$\frac{1}{\sqrt{2\pi}}\,\mathrm{e}^{-\frac{t^2}{2}}$	0	1
χ^2_k $X \sim \chi^2_k$	$[0,+\infty[$	$\sum_{i=1}^k Y_i^2$ where $Y_i \sim \mathcal{N}(0\,;\,1)$	$\frac{1}{\Gamma\left(\frac{k}{2}\right)2^{\frac{k}{2}}}\,t^{\frac{k}{2}-1}\mathrm{e}^{-\frac{t}{2}}$ if $t \geq 0$ and 0 otherwise	k	$2k$
Student $X \sim T_n$	$\mathbb{R}$	$\frac{U}{\sqrt{Y/n}}$ where $U \sim \mathcal{N}(0;1)$ et $Y \sim \chi^2_n$		0 if $n>1$	$\frac{n}{n-2}$ if $n>2$

Remark: The Gamma function (Γ) is defined by:

$$\forall\, x>0, \quad \Gamma(x) = \int_0^{+\infty} t^{x-1}\,\mathrm{e}^{-t}\,\mathrm{d}t$$

One can show that $\forall\, n \in \mathbb{N}, \quad \Gamma(n+1) = n!$ and that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

Estimation

Let $(X_1, X_2, \ldots, X_n)$ be a sample of a random variable X with expected value μ and variance σ^2.

- Empirical mean

$$\overline{X} = \frac{1}{n} \sum_{k=1}^{n} X_k$$

- Empirical variance

$$\Sigma_n^2 = \frac{1}{n} \sum_{k=1}^{n} (X_k - \mu)^2$$

- Corrected empirical variance

$$S_n^2 = \frac{1}{n-1} \sum_{k=1}^{n} (X_k - \overline{X})^2$$

In the trees below, one can find what variable to consider to build confidence intervals depending on the situation.

Expected value

- Normally distributed variables $\mathcal{N}(\mu, \sigma^2)$
  - known variance $\sqrt{n}\,\frac{\overline{X} - \mu}{\sigma} \sim \mathcal{N}(0, 1)$
  - unknown variance $\sqrt{n}\,\frac{\overline{X} - \mu}{S_n} \sim \mathcal{T}_{n-1}$
- Non-normally distributed variables $(n \geqslant 50)$ $\sqrt{n}\,\frac{\overline{X} - \mu}{S_n} \sim_\infty \mathcal{N}(0, 1)$

Variance

- Normally distributed variables $\mathcal{N}(\mu, \sigma^2)$
  - known expected value $n\,\frac{\Sigma_n^2}{\sigma^2} \sim \chi_n^2$
  - unknown expected value $(n-1)\,\frac{S_n^2}{\sigma^2} \sim \chi_{n-1}^2$

Proportion

- Bernoulli variables $\mathcal{B}(p)$
  - $n \geqslant 30$, $np \geqslant 5$ and $np(1-p) \geqslant 5$ $\sqrt{n}\,\frac{\overline{X} - p}{\sqrt{\overline{X}(1 - \overline{X})}} \sim_\infty \mathcal{N}(0, 1)$