

# Usual probability distributions

## Discrete distributions

Name, parameters	$X(\Omega)$	$\mathbb{P}(X = k)$	Expected value	Variance
Bernoulli $X \sim \mathcal{B}(1, p)$	$\{0, 1\}$	$p$ if $k = 1$ $1 - p$ if $k = 0$	$p$	$p(1 - p)$
binomial $X \sim \mathcal{B}(n, p)$	$\llbracket 0, n \rrbracket$	$\binom{n}{k} p^k (1 - p)^{n-k}$	$np$	$np(1 - p)$
hypergeometric $X \sim \mathcal{H}(N, n, m)$	$\subset \llbracket 0, n \rrbracket$	$\frac{\binom{m}{k} \binom{N-m}{n-k}}{\binom{N}{n}}$	$\frac{nm}{N}$	$\frac{n \frac{m}{N} (1 - \frac{n}{N})(N-n)}{N-1}$
geometric $X \sim \mathcal{G}(p)$	$\mathbb{N}^*$	$p(1 - p)^{k-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson $X \sim \mathcal{P}(\lambda)$	$\mathbb{N}$	$e^{-\lambda} \frac{\lambda^k}{k!}$	$\lambda$	$\lambda$

## Continuous distributions

Name, parameters	$X(\Omega)$	Definition	Density $f(t)$	Expected value	Variance
uniform $X \sim \mathcal{U}([a, b])$	$[a, b]$		$\frac{1}{b-a}$ if $t \in [a, b]$ 0 otherwise	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
exponential $X \sim \mathcal{E}(\lambda)$	$[0, +\infty[$		$\lambda e^{-\lambda t}$ if $t \geq 0$ 0 if $t < 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
normal $X \sim \mathcal{N}(\mu, \sigma^2)$	$\mathbb{R}$		$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(t-\mu)^2}{2\sigma^2}}$	$\mu$	$\sigma^2$
standard normal $X \sim \mathcal{N}(0, 1)$	$\mathbb{R}$		$\frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$	0	1
$\chi_k^2$ $X \sim \chi_k^2$	$[0, +\infty[$	$\sum_{i=1}^k Y_i^2$ where $Y_i \sim \mathcal{N}(0; 1)$	$\frac{1}{\Gamma(\frac{k}{2}) 2^{\frac{k}{2}}} t^{\frac{k}{2}-1} e^{-\frac{t}{2}}$ if $t \geq 0$ and 0 otherwise	$k$	$2k$
Student $X \sim T_n$	$\mathbb{R}$	$\frac{U}{\sqrt{Y/n}}$ where $U \sim \mathcal{N}(0; 1)$ et $Y \sim \chi_n^2$		0 if $n > 1$	$\frac{n}{n-2}$ if $n > 2$

REMARK: The Gamma function ( $\Gamma$ ) is defined by:

$$\forall x > 0, \quad \Gamma(x) = \int_0^{+\infty} t^{x-1} e^{-t} dt$$

One can show that  $\forall n \in \mathbb{N}, \quad \Gamma(n+1) = n!$  and that  $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

## Estimation

Let  $(X_1, X_2, \dots, X_n)$  be a sample of a random variable  $X$  with expected value  $\mu$  and variance  $\sigma^2$ .

- Empirical mean

$$\bar{X} = \frac{1}{n} \sum_{k=1}^n X_k$$

- Empirical variance

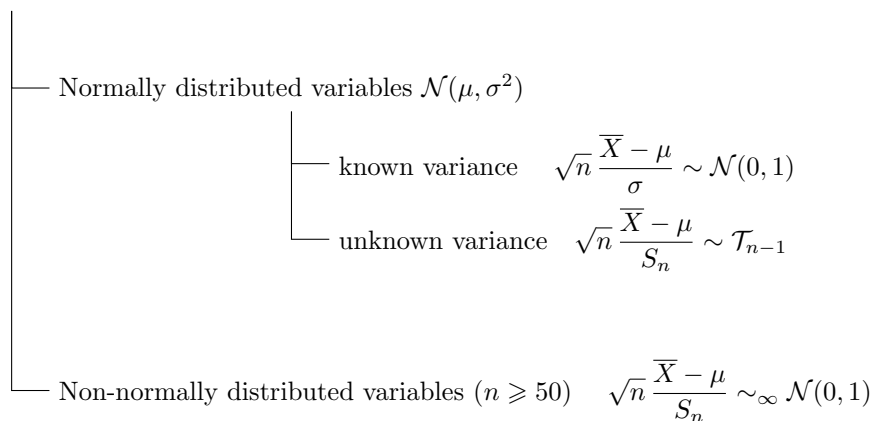
$$\Sigma_n^2 = \frac{1}{n} \sum_{k=1}^n (X_k - \mu)^2$$

- Corrected empirical variance

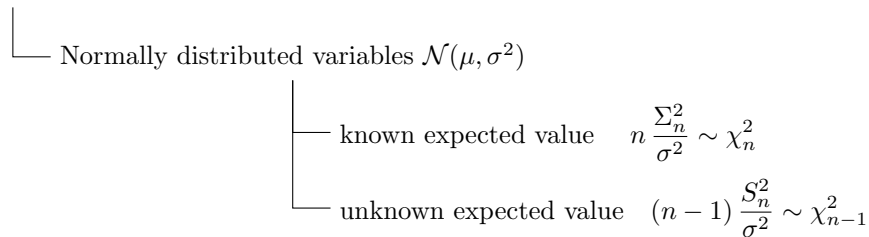
$$S_n^2 = \frac{1}{n-1} \sum_{k=1}^n (X_k - \bar{X})^2$$

In the trees below, one can find what variable to consider to build confidence intervals depending on the situation.

### Expected value



### Variance



### Proportion

