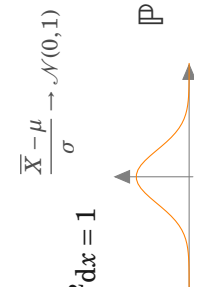


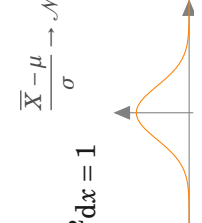
$$\mathbb{E}(\varphi(X)) = \int \varphi(x) d\mathbb{P}_X(x)$$

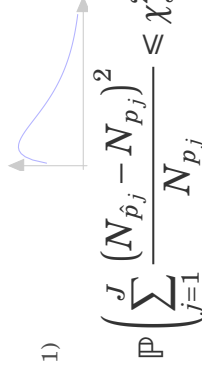
$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-x^2/2} dx = 1$$

$$\mathbb{P}(A) = \sum_{i \in I} \mathbb{P}_{B_i}(A) \mathbb{P}(B_i)$$



$\frac{\bar{X} - \mu}{\sigma} \rightarrow \mathcal{N}(0,1)$





$\mathbb{P}\left(\sum_{j=1}^J \frac{(N_{p_j} - N_{p_j})^2}{N_{p_j}} \leq \chi_{J-1, \alpha}^2\right) \approx 1 - \alpha$

- 1 Read the help file and try out every command you see in it.
- 2 Let $A = [1 \ 2]$ and $B = [3 \ 4]$. Try the following commands in scilab's console : A' , $A * B$, $A * B'$, $A' * B$, $A .* B$.
- 3 Write a program that takes as an argument a number d and gives the area of a circle of diameter d . Your program will give an error message if the diameter is negative.
- 4 Fibonacci's sequence $(F_n)_n$ is characterized by the fact that every number after the first two is the sum of the two preceding ones :

$$\begin{cases} F_0 = 0, F_1 = 1, \\ \forall n \geq 2, F_n = F_{n-1} + F_{n-2}. \end{cases}$$

Write a program that takes n as an argument and that outputs F_n .
If you're up to it you can try a recursive version!

- 5 Plot the sine and cosine functions on the same graphe with different colors and add a legend.
- 6 Let $f : \mathbb{N}^* \rightarrow \mathbb{N}$ defined for all $n \in \mathbb{N}^*$ by :

$$f(n) = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ 3n + 1 & \text{otherwise.} \end{cases}$$

The Collatz conjecture says that given any positive integer n , the sequence $(a_n)_n$ defined by $a_0 = n$ and $a_{n+1} = f(a_n)$ will eventually reach the number 1. Check that this conjecture is true for all integers between 1 and 1000.

- 7 Plot a "large number" of points with random coordinates using the `rand` function. Do they look uniformly distributed on the unit square? Compute the proportion of points that are below the line $\mathcal{D} : y = x$.
- 8 Using the `grand` function with the 'uin' distribution, create a game that :
 - randomly selects a number between 0 and 100
 - asks the user to guess the number (using the `input` function)

- tells the user whether his guess is greater or lesser than the answer and lets him play again until he finds it
- outputs the number of trials when the game is over

9 The middle-square method, suggested by John von Neumann in 1946, is a well-known pseudo-random number generator. Take a number x_0 , compute x_0^2 and let x_1 be the digits in the middle of x_0^2 . Repeating this procedure gives a list of (pseudo)-random numbers. If you work with 10-digits numbers, then divide each of them by 10^{10} and you will have a list of (pseudo)-random numbers between 0 and 1. In order to complete this exercise, you can use the function below :

```
function [x] = middle(num,k)
    //Extracts k digits from the middle of number num
    format(24)
    s = string(num)
    n = length(s)
    deb = round((n-k)/2)
    v = [deb :deb+k-1]
    x = part(s,v)
endfunction
```

1. Write a neumann program following this procedure.
2. Answer exercise 7 with your neumann function instead of `rand`.

10 Using a Riemann sum, give an approximation of $\int_0^1 x dx$.