

In order to answer exercises 1 to 4, you should start with  $N = 1$  and then adapt your program to  $N \in \mathbb{N}^*$ . The "normal" way would be to use a for loop, but the "scilab" way is to use arrays and functions such as `cumsum` and/or `sum`.

**1** Using only the `rand` function, create a function `bernoulli(p,N)` that outputs the result of  $N$  independent Bernoulli experiments with parameter  $p$ .

**2** Using only the `rand` function, create a function `binom(n,p,N)` that outputs the result of  $N$  independent binomial experiments with parameters  $n$  and  $p$ .

**3** Using only the `rand` function, create a function `geometric(p,N)` that outputs the result of  $N$  independent geometric experiments with parameter  $p$ .

**4** Using only the `rand` function, create a function `die(N)` that outputs the result of  $N$  independent fair six-sided die rolls.

**5** Write a program that will play tic-tac-toe against you by choosing its moves randomly. You can use the `input` function and a `matrix` to display the gameboard.

**6** *Hint : `help tabul`.* When rolling two dice, the probability that the sum will be equal to 7 is 1/6.

1. Using the `die` function from exercise 4, simulate 10,000 sums of two dice rolls.
2. Compute the frequency of 7 : is this result surprising?
3. Give an approximation of the probability mass function for the sum of two independent dice rolls.
4. Compute the distribution and check that your approximation is good.

**7** A deck of 52 cards can be represented by the integers between 1 and 52.

1. Write a program that randomly picks 5 "cards" in the set  $\llbracket 1;52 \rrbracket$ .

2. Repeat the experiment 100,000 times and compute the frequency of "hands" with one queen and two kings.
3. Compute the probability that, when picking 5 random cards in a deck of 52 cards, there will be exactly one queen and two kings.

**8** The law of large numbers states that, in layman's terms, if you repeat a random experiment an infinite number of times, then the average result will be equal to the expected value of the underlying random variable.

1. Simulate 100,000 Bernoulli experiments with a parameter  $p$  of your choice.
2. Compute the corresponding average value.
3. Compare it to the expected value of the Bernoulli distribution.

Repeat questions 1 to 3 with the other distributions from the course.

**9** Consider the following experiment ; an urn contains 5 marbles, 4 blue ones and 1 green one. Someone picks a marble at random : if it's blue, he puts 4 blue marbles back, otherwise he puts back 2 green ones. After 100 trials, the person wins \$ $n$  where  $n$  is the proportion of green marbles.

1. Using the `grand` function with the '`uin`' parameter, write a function that takes a number of green and blue marbles as an argument, simulates the experiment and then outputs the new number of green and blue marbles.
2. Write a function that "plays" 100 times.
3. Simulate a "large number" of games (for example 100,000) and compute the average winnings.

**10** Take a random exercise from the first page of the exercise sheet and write a `scilab` function that simulates the corresponding experiment.

$$\mathbb{E}(\varphi(X)) = \int \varphi(x) d\mathbb{P}_X(x)$$

$$\binom{n}{k} p^k (1-p)^{n-k}$$

$$\mathbb{P}(A) = \sum_{i \in I} \mathbb{P}_{B_i}(A) \mathbb{P}(B_i)$$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-x^2/2} dx = 1$$

$$\frac{\bar{X} - \mu}{\sigma} \rightarrow \mathcal{N}(0,1)$$

$$\mathbb{P} \left( \sum_{j=1}^J \frac{(N \hat{p}_j - N p_j)^2}{N p_j} \leq \chi_{J-1, \alpha}^2 \right) \simeq 1 - \alpha$$